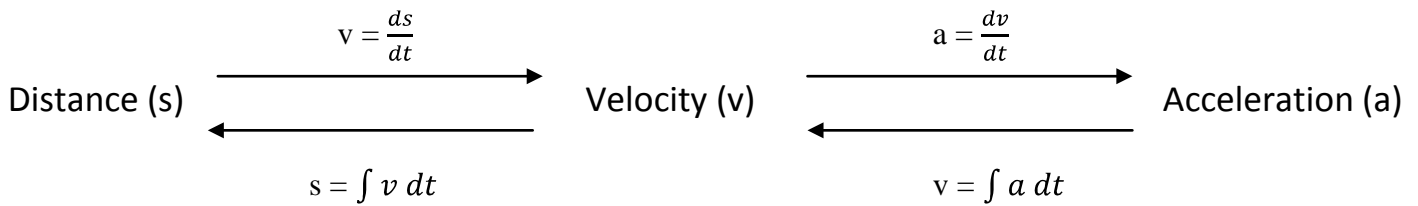


Rates of Change involving KINEMATICS

Kinematics = study of motion along a straight-line path. The relationship between distance, velocity, and acceleration is displayed below.



Conventions

Several conventions hold in kinematics problems.

1) **Initial values** apply when time is 0, i.e. when motion started.

2) Zero values

- a) Distance = 0 means object is at the **origin**
- b) Velocity = 0 means the object is **stationary**.
- c) Acceleration = 0 means the object is travelling at **constant speed**.

3) Positive values

- a) Distance > 0 means object is **above** or to the **right** of the origin.
- b) Velocity > 0 means object is travelling **forwards** or **upwards**.
- c) Acceleration > 0 means object is **speeding up**.

4) Negative values

- a) Distance < 0 means object is **below** or to the **left** of the origin.
- b) Velocity < 0 means object is travelling **backwards** or **downwards**.
- c) Acceleration < 0 means object is **slowing down**.

5) $\int_{t_1}^{t_2} v(t) dt$ gives the distance travelled in between t_1 and t_2 seconds.

Examples

1. A stone is thrown vertically upwards into the air. Its height in metres above the ground is given by $s(t) = 30t - 5t^2$.

- a) Calculate the initial velocity
- b) Calculate the maximum height reached by the object
- c) When does the object return to the ground?

a) **Velocity** $v(t) = s'(t) = 30 - 10t$
 Initial velocity $v(0) = 30 - 10(0) = 30 \text{ m/s}$

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b) Max height occurs when object stops travelling upwards and starts to fall, and this is where its velocity is 0, i.e. solve $v(t) = 0$: $\Rightarrow 30 - 10t = 0 \Rightarrow t = 3$ secs
Therefore, maximum height is reached at $t = 3$ secs:
 $s(3) = 30(3) - 5(3)^2 = 45$ metres.

c) Object reaches ground when it's distance from the ground is 0, so solve $s(t) = 0$:
 $30t - 5t^2 = 0 \Rightarrow 5t(6 - t) = 0 \Rightarrow t = 0$ and $t = 6$.
Therefore the object returns to the ground at $t = 6$ seconds.

2. The velocity (in m/s) of an object t seconds after it started from the origin is given by $v(t) = 3t^2 - 14t - 5$. The object is travelling in a straight line in an East-West direction.

- a) Give the formulae for the acceleration and distance after t seconds.
- b) Calculate the initial velocity. In which direction is the object moving?
- c) When is the object at rest?
- d) Calculate the minimum velocity. Interpret this answer.
- e) How far did the object travel in the eighth second?

a) Distance, $s = \int v \, dt = t^3 - 7t^2 - 5t + c$.
Object started from origin, so $s = 0$ when $t = 0$, giving $c = 0$.
Therefore $s = t^3 - 7t^2 - 5t$

Acceleration, $a = \frac{dv}{dt} = 6t - 14$.

b) Initial velocity, $v(0) = 3(0)^2 - 14(0) - 5 = -5$ m/s. Object is travelling East.

c) Object is at rest when $v(t) = 0$:
 $3t^2 - 14t - 5 = 0 \Rightarrow (3t + 1)(t - 5) = 0 \Rightarrow t = -\frac{1}{3}, t = 5$

d) Minimum velocity occurs when its derivative is 0 i.e. when $a(t) = 0$:
 $6t - 14 = 0 \Rightarrow t = 2\frac{1}{3}$ secs.
Min velocity, $v\left(2\frac{1}{3}\right) = 3\left(2\frac{1}{3}\right)^2 - 14\left(2\frac{1}{3}\right) - 5 = -21\frac{1}{3}$ m/s occurs when object is travelling East.

e) Distance travelled in the eighth second, $\int_7^8 v \, dt = \int_7^8 3t^2 - 14t - 5 \, dt$
 $= \left[\frac{3t^3}{3} - \frac{14t^2}{2} - 5t\right]_7^8 = 59$ metres.